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Cubic Frequency-Shift Filtering for Cochannel Interference Removal[1]

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Abstract

It is shown that cochannel interference removal from digital quadrature-amplitude-modulated signals without the introduction of signal distortion can be accomplished by using linear-plus-cubic frequency-shift filtering. The theory of higher-order cyclostationary signals is used to explain how this can be done, and the results of simulations that quantify the performance of the method are presented.

1 Introduction

One of the fundamental difficulties encountered in communication system design is the problem of cochannel interference. When the interfering signal is narrowband with respect to the signal of interest (SOI), or overlaps the SOI only over a small portion of its allotted band of frequencies, excision methods (i.e., narrowband filtering) can sometimes be used to mitigate the interferer's effects while introducing a level of distortion into the SOI that is either tolerable or correctable by other means. However, when this distortion is unacceptable, as it usually is when the interfering signal overlaps the SOI over a sufficiently large portion of its allotted band, other means of interference mitigation must be sought. This difficulty arises because the communication signals involved are usually modeled as stationary stochastic processes, for which linear time-invariant (LTI) filtering is appropriate and, in some cases, optimal from a minimum-mean-squared-error viewpoint. Nevertheless, LTI filtering cannot *separate* spectrally overlapping signals.

Over the past several years it has been established that many communication signals are more appropriately modeled as cyclostationary stochastic processes in the sense that superior signal processing methods, such as detectors and estimators, can be derived from this model [1, 2]. These methods are superior in that they can deliver more accurate estimates for the same amount of data, and in some cases they can solve problems that cannot be solved by using the simpler stationary model. A signal $x(t)$ is called *second-order cyclostationary* if there exists a stable time-invariant quadratic transformation such that the output of the transformation with $x(t)$ at the input contains at least one finite-strength additive sine-wave component with nonzero frequency α [1]. This property of cyclostationary signals is called the *sine-wave regeneration property*. Equivalently, a signal is second-order cyclostationary if it contains pairs of spectral components—separated in frequency by a nonzero amount α—that are temporally correlated. This property of cyclostationary signals is called the *spectral correlation property*. The frequency separations and the frequencies of the generated sine waves are specified by the same set of numbers $\{\alpha\}$, which are called *cycle frequencies*.

In particular, it has recently been demonstrated that by using linear periodically (or almost periodically) time-variant filtering, cochannel interference can, in principle, be completely removed from a second-order cyclostationary signal—without introducing any distortion into the signal [3]. A linear (almost) periodically time-variant system is also referred to as a FREquency-SHift (FRESH)

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filter because the system output can be represented as the sum of filtered and frequency-shifted versions of the input. For signals that do not exhibit second-order cyclostationarity (SOCS), linear FRESH filtering degenerates to LTI filtering.

With the increasing use of the radio spectrum, there has been a corresponding increase in the use of spectrally efficient signals (e.g., partial response and some digital quadrature-amplitude-modulated [QAM] signals). These signals exhibit weak or no SOCS, and therefore linear FRESH filtering provides little or no benefit over LTI filtering. However, these signals do exhibit *higher-order cyclostationarity* (HOCS) [4, 5], and this property can be exploited to yield a generalization of linear FRESH filtering to nonlinear FRESH filtering. In this paper, we propose a nonlinear method of removing cochannel interference from signals that do not exhibit SOCS. The method consists of linearly and cubically transforming the received data, frequency shifting the results, and summing to yield an estimate of the SOI: linear-plus-cubic (LPC) FRESH filtering.

The remainder of the paper is organized as follows. In Section 2 we provide the mathematical background on HOCS that is necessary to understand the LPC method, in Section 3 we describe the method, and in Section 4 we present results of a computer study of the method that give an indication of its potential. Finally, in Section 5 we draw conclusions and outline appropriate follow-on research problems.

2 Cyclostationarity

The theory of cyclostationary signals can be developed in either the stochastic-process framework [2, 6] or the time-series framework [1, 4, 6, 7]. We use the latter here. Consider a time-series $x(t)$ defined for all time t. We introduce some quantities that are nonlinear transformations of this time-series. These quantities, whose mathematical existence is assumed, are obtained by using the multiple-sine-wave-extraction operator $\hat{E}^{\{\alpha\}}\{\cdot\}$, which extracts the almost periodic component of its argument (and is analogous to the stochastic expectation operation) [2, 7]. That is, $\hat{E}^{\{\alpha\}}\{\cdot\}$ extracts the sum of all finite-strength additive sine waves present in its argument.

The *nth-order temporal moment function* (TMF) for $x(t)$ is defined by

$$R_x(t,\underline{\tau})_n \triangleq \hat{E}^{\{\alpha\}}\left\{\prod_{j=1}^{n} x^{(*)_j}(t+\tau_j)\right\} = \sum_{\alpha_n} R_x^{\alpha}(\underline{\tau})_n e^{-i2\pi\alpha t}, \tag{1}$$

where $(*)_j$ denotes the optional conjugation of the factor $x(t+\tau_j)$. The complex-valued strength of each of the sine-wave components of the TMF is called a *nth-order cyclic temporal moment function*, and can be computed in the following way

$$R_x^{\alpha}(\underline{\tau})_n = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\prod_{j=1}^{n} x^{(*)_j}(t+\tau_j)e^{-i2\pi\alpha t}\,dt = \left\langle \prod_{j=1}^{n} x^{(*)_j}(t+\tau_j)e^{i2\pi\alpha t}\right\rangle, \tag{2}$$

where the angle brackets $\langle\cdot\rangle$ are used to denote the infinite-time-averaging operation.

A signal $x(t)$ for which there is at least one nonzero value of α for which $R_x^{\alpha}(\underline{\tau})_n \not\equiv 0$ in the sum in (1) are said to exhibit *nth-order cyclostationarity.* For $n > 2$, $x(t)$ is said to exhibit higher-order cyclostationarity. An interesting case is that in which $x(t)$ exhibits cyclostationarity for order n but not for any order smaller than n. Examples of this kind of signal for $n = 4$ are members of the class of partial-response signals [8]. Other interesting signals are those that exhibit weak cyclostationarity for orders less than n, but strong cyclostationarity for order n. Examples of these signals for $n = 4$ are bandwidth-efficient digital QAM signals with excess bandwidth on the order of 10% (or less).

In the special case of $n = 2$, $\tau_1 = \tau/2$, and $\tau_2 = -\tau/2$, the CTMF coincides with the *cyclic autocorrelation function*, which is typically denoted by $R_x^\alpha(\tau)$ [1]. Linear FRESH filtering can be understood by interpreting the cyclic autocorrelation as the cross correlation between a signal $s(t+\tau/2)$ and a time- and frequency-shifted version of itself $s(t-\tau/2)e^{i2\pi\alpha t}$:

$$R_s^\alpha(\tau) = \left\langle s(t+\tau/2)\left[s(t-\tau/2)e^{i2\pi\alpha t}\right]^*\right\rangle.$$

Let $x(t) = s(t) + m(t)$, where $s(t)$ is the SOI and $m(t)$ is the SNOI plus noise. The idea is to estimate the SOI $s(t)$ in the data $x(t)$ by using both time- and frequency-shifted versions of $x(t)$.

The motivation for cubic FRESH filtering is understood by interpreting the fourth-order CTMF as the cross correlation between a signal $s(t+\tau_1)$ and a cubically transformed and frequency-shifted version of itself $\prod_{j=2}^4 s(t+\tau_j)e^{i2\pi\alpha t}$:

$$R_s^\alpha(\underline{\tau})_4 = \left\langle s(t+\tau_1)\left[\prod_{j=2}^{4} s(t+\tau_j)e^{i2\pi\alpha t}\right]^*\right\rangle.$$

The idea is to estimate the SOI $s(t)$ in the data $x(t)$ by using frequency-shifted cubically transformed versions of $x(t)$.

3 Linear-Plus-Cubic Frequency-Shift Filtering

For a real-valued input signal $x(t)$ the output $y(t)$ of an LPC system is given by the following relation

$$\begin{aligned} y(t) = &\sum_\eta \left[\int_{-\infty}^{\infty} h_\eta(\tau)x(t-\tau)e^{i2\pi\eta t}\,d\tau\right] \\ &+\sum_\alpha \left[\int_{-\infty}^{\infty} g_\alpha(\underline{\tau})x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)e^{i2\pi\alpha t}\,d\underline{\tau}\right], \quad \underline{\tau} = [\tau_1\ \tau_2\ \tau_3]. \end{aligned} \tag{3}$$

The sum over the frequency parameters $\{\eta\}$ describes a linear FRESH filter, the sum over the frequency parameters $\{\alpha\}$ describes a cubic FRESH filter, and together they describe a linear-plus-cubic FRESH filter.

The problem of finding the functions $\{h_\eta(\cdot)\}$ and $\{g_\alpha(\cdot)\}$ such that $y(t)$ is the minimum-mean-squared-error estimate of $s(t)$,

$$\min_{\{h_\eta, g_\alpha\}} \left\langle |y(t) - s(t)|^2\right\rangle$$

results in a set of multidimensional integral equations that are difficult to solve even for specific simple models for $s(t)$ and $m(t)$. However, the form of the integral equations does allow specification of the parameter sets $\{\eta\}$ and $\{\alpha\}$, which are not limited to the second- and fourth-order cycle frequencies of $x(t)$, respectively (cf. [3] for the case of linear FRESH filters). In fact, both sets of parameters are specified by the same frequencies, which are all integer-coefficient linear combinations of the set of second-, fourth-, and sixth-order cycle frequencies for $x(t)$, which itself consists of the cycle frequencies for $s(t)$, $i(t)$, and $n(t)$, for orders 2, 4, and 6, as well as the sums and differences of their cycle frequencies for orders 2 and 4 [4, 5].

In this paper, we focus on the simpler problem of least-squares estimation of the kernels $\{h_\eta(\cdot)\}$ and $\{g_\alpha(\cdot)\}$ for a given finite-duration data set consisting of a SOI, a SNOI, and noise. A discrete-time version of the nonlinear system (3) is given by

$$y(t) = \sum_\eta \left[\sum_u h_\eta(u)x(t-u)e^{i2\pi\eta t}\right] + \sum_\alpha \left[\sum_{\underline{u}} g_\alpha(\underline{u})x(t-u_1)x(t-u_2)x(t-u_3)e^{i2\pi\alpha t}\right]. \tag{4}$$

A computer study was conducted to determine the filter coefficients $h_\eta(\cdot)$ and $g_\alpha(\cdot)$ given a data set $x(t)$ consisting of a SOI $s(t)$ that is corrupted by an interferer $i(t)$ (signal not of interest [SNOI]) and white Gaussian noise $n(t)$: $x(t) = s(t)+i(t)+n(t)$. Because of the rather severe computational burden associated with estimation of a set of multidimensional functions, it is necessary to limit the domain over which the $g_\alpha(\cdot)$ are to be estimated. It is also of some practical interest to determine the performance that can be attributed to these limited-domain portions of the functions $g_\alpha(\cdot)$ because this might lead to computationally simpler LPC filters that still yield acceptable performance.

To this end, consider (4) with the sum over u_1 and u_2 restricted to the sets U_α:

$$\begin{aligned} y(t) = & \sum_\eta \left[\sum_u h_\eta(u)x(t-u)e^{i2\pi\eta t}\right] \\ & + \sum_\alpha \left[\sum_{(u_1,u_2)\in U_\alpha} \left\{x(t-u_1)x(t-u_2)\left[\sum_{u_3} g_\alpha(\underline{u})x(t-u_3)e^{i2\pi\alpha t}\right]\right\}\right]. \end{aligned} \tag{5}$$

From this formulation, it is clear that the LPC system can be specified entirely in terms of one-dimensional filters, that is, in terms of one-dimensional sections of the three-dimensional functions $g_\alpha(\cdot)$. Thus, we seek the values of $g_\alpha(\cdot)$ for various values of (u_1, u_2). For each value of α, each fixed pair $(u_1, u_2) \in U_\alpha$ is said to result in a distinct *path* in the LPC structure. The remaining questions concern the choice of the elements of $\{\eta\}, \{\alpha\}$, and U_α such that the resulting paths contribute maximally to the quality of the output of the LPC structure. This can be done by examining the correlation between the cubic paths of the LPC structure with the SOI. The choices of U_α should be made such that this correlation is largest. Thus, we should choose the lag pairs such that the fourth-order moment corresponding to the SOI is largest. In the case of duobinary signals, the CTMFs peak for $\underline{\tau} = \mathbf{0}$.

4 Computer Simulations

MATLAB was used to solve the following least-squares problem

$$\min_{\{h_\eta, g_\alpha\}} \sum_{t=1}^{T} [y(t) - s(t)]^2, \tag{6}$$

where $y(t)$ is the output of the LPC filter (5) that with input $x(t)$ and T is the data-record length, for various values of $\{\eta\}$, $\{\alpha\}$, U_α, and the filter-path lengths. The SOI $s(t)$ (and the SNOI $i(t)$) is a duobinary-coded pulse-amplitude modulated signal, which is defined by

$$s(t) = \sum_{m=-\infty}^{\infty} a_m p(t + mT_0 + t_0), \qquad P(f) = \begin{cases} T_0(1 + e^{-i2\pi f T_0}), & |f| \le 1/2T_0 \\ 0, & |f| > 1/2T_0, \end{cases}$$

where $P(f)$ is the Fourier transform of the pulse $p(t)$, $\{a_m\}$ is a random binary symbol sequence, and t_0 is a timing offset. The noise $n(t)$ is white and Gaussian with power in the receiver band that is 20dB below the total SOI power. None of the signals $s(t), i(t)$, or $n(t)$ exhibit SOCS, but both $s(t)$ and $i(t)$ exhibit fourth-order cyclostationarity at their symbol rates, which are $1/T_0$ and $1/T_1$, respectively. In all the simulations, the SOI and SNOI power levels are equal.

Two cochannel interferences are considered: wideband ($T_1 = 11$) and narrowband ($T_1 = 44$). Minimum mean-squared errors (MSEs) for the wideband interferer are shown in Table 1 for $\{\eta\} = \{0, 1/T_0\}$ and several choices of $\{\alpha\}$. Zero and the symbol-rate cycle frequencies of the SOI are

Case	$\{\eta\}$	$\{\alpha\}$	u_1, u_2	No. of Paths	MSE
1	0	$\{\}$	—	1	0.88
2	$0, \pm 1/T_0$	0	(0,0)	4	0.65
3	$0, \pm 1/T_0$	$\pm 1/T_0$	(0,0)	5	0.61
4	$0, \pm 1/T_0$	$0, \pm 1/T_0$	(0,0)	6	0.55
5	$0, \pm 1/T_0$	$0, \pm 1/T_0, \pm 1/T_1$	(0,0)	8	0.47
6	$0, \pm 1/T_0$	$0, \pm 1/T_0, \pm 1/T_1, \pm(1/T_0 - 1/T_1)$	(0,0)	10	0.36

Table 1: Estimation errors for an LPC FRESH filter: wideband interferer (see text).

Case	$\{\eta\}$	$\{\alpha\}$	u_1, u_2	No. of Paths	MSE
Filter Length = 32					
1	0	$\{\}$	—	1	0.44
2	0	0	(0,0), (0,1), (0,2)	4	0.39
3	0	$\pm 1/T_0$	(0,0), (0,1), (0,2)	7	0.38
4	0	$0, \pm 1/T_0$	(0,0), (0,1), (0,2)	10	0.33
Filter Length = 64					
5	0	$\{\}$	—	1	0.32
6	0	0	(0,0), (0,1)	3	0.27
7	0	$\pm 1/T_0$	(0,0), (0,1)	5	0.25
8	0	$0, \pm 1/T_0$	(0,0), (0,1)	7	0.20
9	0	$\{\}$	—	1	0.32
10	$0, \pm 1/T_0$	0	(0,0), (0,1)	5	0.26
11	$0, \pm 1/T_0$	$\pm 1/T_0$	(0,0), (0,1)	7	0.23
12	$0, \pm 1/T_0$	$0, \pm 1/T_0$	(0,0), (0,1)	9	0.19

Table 2: Estimation errors for an LPC FRESH filter: narrowband interferer (see text).

assumed to be the most important frequency shifts, but substantial improvements would likely occur with the addition of more frequency shifts into both the linear and cubic paths. The best performance for a wideband SNOI is Case 6, which corresponds to a reduction in MSE of 60% with respect to the LTI LPC system (Case 1).

Similar results are shown for the narrowband interferer in Table 2. For this SNOI, we assume that the SNOI is stationary, and so we do not include its cycle frequencies in the linear or cubic paths. However, the performance of the method would increase if these were taken into account. The best performance for a narrowband SNOI is Case 12, which corresponds to a reduction in MSE of 41% with respect to the LTI LPC system (Case 5).

5 Conclusions

The general LPC system is difficult to analyze and simulate, so only suboptimal computationally simpler versions of the general system have been studied so far. These simpler systems are characterized by the number of linear and cubic paths from the input to the output. The mean-squared-error in the estimate of a signal corrupted by cochannel interference, obtained by a least-squares technique, is shown to decrease with an increase in the total number of paths used. The results of this initial study suggest that the LPC filter structure holds some promise for removing cochannel interference from a bandwidth-efficient signal. An important theoretical result to be obtained in the future is the solution of the minimum-mean-squared-error design equations that specify the system parameters. A practical problem to be solved is finding rules-of-thumb for choosing good subsets of the system parameters.

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